q, heat flux density; Ω, region occupied by the liquid; $\vec{r}$, distance from an arbitrary point in the liquid to the body; $\Omega_t = \Omega \times [0, t]$; g, acceleration due to gravity; ρ, λ, a, the liquid density, thermal conductivity, and thermal diffusivity; σ, the integral emissivity; $N = 4\sigma T_\infty^3/\lambda U$; $L_2(\Omega_t)$, Hilbert space; $\overset{\circ}{W}{}_2^{1,1}(\Omega_t)$, $\overset{\circ}{W}{}_{2;\exp(-\varphi)}^{1,1}(\Omega_t)$, Sobolev space.

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TWO-DIMENSIONAL TEMPERATURE DISTRIBUTION IN A CERAMIC-BASED ELECTRODE

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UDC 536.24.02

A study has been made of the thermal processes in the electrode units in an MHD channel; generalized relationships between the geometrical parameters of the blocks and the parameters of the working body have been derived.

Much attention is now being given to large MHD systems containing sectional ceramic electrodes for use in fully commercial or pilot MHD stations [1]. The viability and working lives of such systems are largely determined by the thermal conditions in the electrode blocks.

There are several papers on the temperature distributions in such blocks; for instance, temperature distributions have been determined [1, 2] for ceramic electrode modules enclosed in metal matrices. Estimates have been made [1] of the maximum temperature in a module and the time needed to reach the steady thermal state for blocks of various sizes and various heat-flux levels at the MHD channel wall.

However, most studies [1-5] are based on solving the thermal-conduction equations subject to major simplifications (constant temperature in the metal matrix, constant thermophysical parameters of the electrode materials, etc.), which substantially restrict the applicability of the results to viability evaluation.

§1. Figure 1a, b shows some typical electrode schemes based on ceramic modules made of zirconium dioxide ZrO_2 [2, 3]. A ceramic module is enclosed in a metal cooling matrix, while the electrical insulation is provided by plates of Al_2O_3 or MgO.

The metal matrix in Fig. 1a performs two functions: it cools the ceramic element and also handles the current through the upper parts of the metal edges. Since ZrO_2 ceramic is of fairly high electrical conductivity ($\sigma >$ 10-20 mho/m) only at high temperatures ($T \geqslant$ 1100-1200°K) [1], the edge of the matrix must be made of heat-resisting steel.

In Fig. 1b, the current is carried by high-temperature metal grid or plate embedded in the ceramic element, which reduces the severity of the working conditions for the metal cooling edges and allows one to make the matrix of a metal of high thermal conductivity such as copper.

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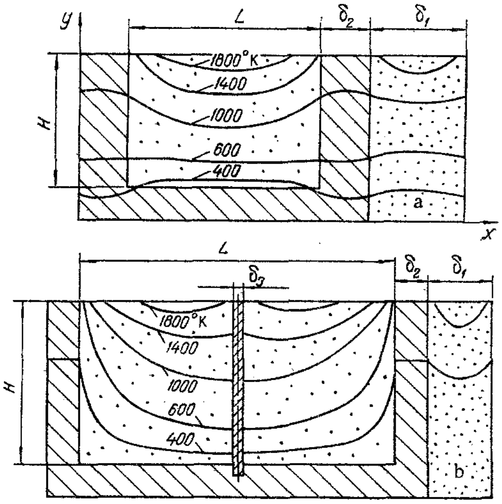

Fig. 1. Electrode blocks with ceramic modules based on ZrO_2: a) 1Kh18N10T matrix; b) copper matrix.

§2. Theoretical investigations on temperature distributions in electrodes must be based on a model that incorporates fully the electrode design and the temperature dependence of the thermophysical parameters, as well as the actual heat-transfer conditions in an MHD channel.

The working schemes of Fig. 1a, b are based on the models consisting of rectangular elements differing in thermophysical parameters; it is assumed that the conditions for continuity in the temperature T and heat flux q are obeyed at the internal boundaries:

$$T_+ = T_- \text{ and } \lambda_+ \frac{\partial T_+}{\partial n} = \lambda_- \frac{\partial T_-}{\partial n}. \tag{1}$$

Here λ is the thermal conductivity; n is the direction of the normal to the interface; and the subscripts "+" and "—" denote, respectively, the quantities to the right and left of the interface.

It is assumed that the temperature and heat flux at the external side faces satisfy the conditions of (1) for each section y = const; this is approximately so in an actual MHD device if the heat-transfer conditions vary only slightly along the axis.

It is assumed that the heat flux from the hot gas at the upper boundary is governed by turbulent convective transfer; it has been shown [4] that this assumption is closely met for MHD devices based on combustion products from current chemical fuels.

It is assumed that the lower boundary (y = 0) is thermally insulated ($\partial T/\partial y = 0$) or else has a specified temperature; these assumptions correspond to two limiting cases: heat accumulation and rapid forced cooling of the metal matrix.

It can be shown that the following estimate applies for the electrodes in a large-scale MHD device:

$$\frac{\int_0^H \frac{j^2}{\sigma_c} dy}{\alpha_0 (T_\infty - T_w)} \sim \frac{\sigma_g}{\sigma_c} \left(\frac{1-k}{k}\right) \frac{\eta}{c_f \left(1 - \frac{T_w}{T_\infty}\right)} \frac{H_c}{L_c} \ll 1. \tag{2}$$

Here σ_g and σ_c are the conductivity of the gas in the MHD channel and the conductivity of the ceramic module; k is the load factor; T_∞ and T_W are, respectively, the stagnation temperature of the gas flow and the temperature of the electrode wall; c_f is the coefficient of friction; η is the energy-conversion factor for the MHD channel; H_c is the height of the module; L_c is the length of the channel; and α_0 is the heat-transfer coefficient.

It follows from (2) that the effects of heat produced by the current flowing in the module can be neglected in relation to the temperature distribution in the electrodes; the above assumptions then reduce the problem to that of two-dimensional nonlinear nonstationary thermal conduction with discontinuous coefficients:

$$\rho(x, y, T)\, C(x, y, T) \frac{\partial T}{\partial t} = \frac{\partial}{\partial x}\left(\lambda(x, y, T) \frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(\lambda(x, y, T) \frac{\partial T}{\partial y}\right), \tag{3}$$

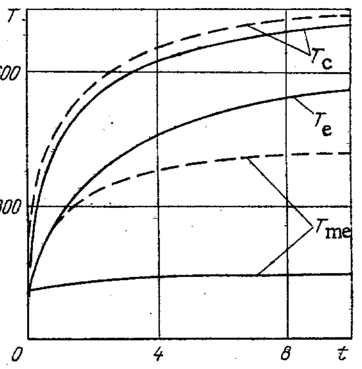

Fig. 2

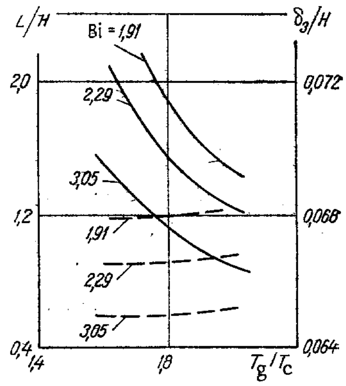

Fig. 3

Fig. 2. Time course of temperature in electrode-block elements during startup; solid curves) block with copper matrix; dashed curves) block with steel matrix. T in °K; t in sec.

Fig. 3. Relative ceramic module size L/H and relative grid size δ_3/H as functions of the relative temperature T_g/T_c and Biot criterion Bi for block with a copper matrix [solid curves) L/H; dashed curves) δ_3/H].

where ρ is the density; C is the thermal capacity; and λ is the thermal conductivity.

The boundary conditions at the outer boundaries of the region are put as

$$T(0,\ y) = T(L,\ y);\ \frac{\partial T(0,\ y)}{\partial x} = \frac{\partial T(L,\ y)}{\partial x}, \tag{4}$$

$$\lambda\,\frac{\partial T}{\partial y} = \alpha\,(T_g - T)\ \text{ at }\ y = H,\ \ \nu T + (1-\nu)\,\frac{\partial T}{\partial y} = \nu T_l\ \text{ at }\ y = 0. \tag{5), (6}$$

Here $\alpha = \alpha_0/(1 + \mathrm{Bi}_{sl})$ is the reduced heat-transfer coefficient due to the presence of a slag film on the electrode; $\mathrm{Bi}_{sl} = \alpha\,(\delta_{sl}/\lambda_{sl})$ is the Biot number for the slag film of thickness δ_{sl}; T_g is the gas temperature; the subscript $\nu = 1$ corresponds to a cooled module; and $\nu = 0$ corresponds to a module working with heat accumulation.

The initial condition is that the temperature T_l is initially the same throughout the module, namely, when $t \le 0$.

§3. Finite-difference methods can be used efficiently to solve (3) if the range of continuous variation in the arguments is replaced by a net having the nodal coordinates $x_i = i \cdot h$, $y_j = j \cdot h_1$ ($i = 0, 1, 2, \ldots, M$, $j = 0, 1, 2, \ldots, N$), where h and h_1 are, respectively, the scales of the steps along the x and y axes.

The numerical solution was derived by means of an inexplicit approximation to (3) of the form

$$\frac{T^{k+1} - T^k}{\tau} = \frac{1}{h^2}\left[a_{i-1/2}T_{i-1}^{k+1} - (a_{i-1/2} + a_{i+1/2})\,T_i^{k+1} + a_{i+1/2}T_{i+1}^{k+1}\right]$$
$$+ \frac{1}{h_1^2}\left[a_{j-1/2}T_{j-1}^{k+1} - (a_{j-1/2} + a_{j+1/2})\,T_j^{k+1} + a_{j+1/2}T_{j+1}^{k+1}\right], \tag{7}$$
$$0 \leqslant i \leqslant M,\ 0 \leqslant j \leqslant N,$$

where τ is the time step, $a = \lambda/\rho C$ is the thermal diffusivity, and k is the number of the time step.

The boundary conditions (5) and (6) were approximated to the second degree [6] by means of the following: for y = 0

$$T_0 = (1-\nu)\,\frac{\gamma_1 T_1 + \dfrac{1}{2a_0}\,T_0^k + \dfrac{\gamma}{2\lambda_0}\,(\lambda_{i-1/2}T_{i-1} + \lambda_{i+1/2}T_{i+1})}{\gamma_1 + \gamma + \dfrac{1}{2a_0}} + \nu T_l$$

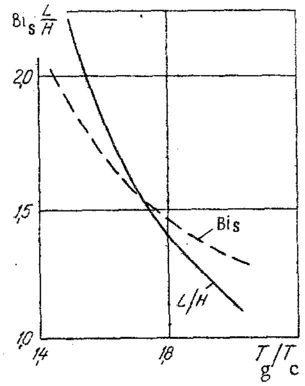

Fig. 4. Major parameters of ceramic module in relation to relative temperature T_g/T_c for a block with a steel matrix.

and for y = H

$$T_N = \left[\gamma_1 T_{N-1} + \frac{1}{2a_N} T_N^k + \frac{\tau\cdot\alpha}{h_1\lambda_N} T_g \left(1 + \frac{\lambda_N - \lambda_{N-1}}{2\lambda_N}\right) + \frac{\gamma}{2\lambda_N}\right.$$

$$\left.\times (\lambda_{i-1/2}T_{i-1} + \lambda_{i+1/2}T_{i+1})\right]\left[\gamma_1 + \gamma + \frac{1}{2a_N} + \frac{\tau\cdot\alpha}{h_1\lambda_N}\left(1 + \frac{\lambda_N - \lambda_{N-1}}{2\lambda_N}\right)\right]^{-1}, \quad (8)$$

where $\gamma = \tau/h^2$; $\gamma_1 = \tau/h_1^2$.

The numerical solution to (7) with (8) was obtained by single fitting for the y axis and iterative fitting for the x axis [6]. The T dependence of λ, C, and ρ was incorporated iteratively for each step τ by reference to the mean temperature $T = (T_{i,j}^k + T_{i,j}^{k+1/2})/2$.

Linear functions were used to approximate the T dependence of λ, ρ, and C for each range of continuous variation:

$$\lambda = \lambda_0 + \beta_\lambda T;\ C = C_0 + \beta_c T \text{ and } \rho = \rho_0 + \beta_\rho T.$$

§4. This method was used in calculating the two-dimensional temperature distributions in these electrode units (Fig. 1a, b); the material of the insulator was taken as alumina Al_2O_3, while the coefficients in the polynomials for the thermophysical parameters were derived from the data of [7, 8].

The number of points taken along the x axis was chosen as M = 20, while the number along the y axis was N = 15. The time step was τ = 0.05 sec. Check calculations were performed with more numerous points, M = 40, N = 15, and τ = 0.02, but they gave no substantial improvement in the accuracy.

Figure 1a shows the steady-state temperature distribution in a cooled electrode block containing a steel matrix.

The effective gas temperature T_g, the heat-transfer coefficient α, and the temperature T_l of the lower boundary of the module were taken, respectively, as 3500°K, 1000 W/m^2°K, and 300°K.

The geometrical parameters of the module δ_1 = 3 mm, δ_2 = 1.5 mm, l = 12 mm, H = 4 mm, L = 6 mm, L/H = 1.5 were selected for the given T_g, α, and T_l in such a way that the maximum temperatures found for the modules (~2000°K) and metal edges (~1200°K) were close to the maximum permissible.

Figure 1a shows that the upper part of the ZrO_2 module had a temperature above 1200°K, which provides high conductivity ($\sigma \geqslant 20$ mho/m) throughout the width.

Calculations were also performed for other thicknesses δ_2 for the cooled edges, and it was found that δ_2 had little effect on the temperature distribution in the module for $\delta_2 > 1.5$ mm, which means that if this condition is met, one can select the sizes of the cooling edges from other considerations.

Figure 1b shows the temperature distribution in a block containing a copper matrix and a metal current-collecting edge made of VZh-98 heat-resisting steel.

The geometrical parameters of the major elements were taken as δ_1 = 2 mm, δ_2 = 1 mm; δ_3 = 0.3 mm, l = 14 mm, L = 10 mm and L/H = 2 on the basis that the given values T_g = 3500°K, α = 1000 W/m^2·°K and T_l = 300°K would result in maximum temperatures for the elements close to the maximum acceptable (temperature of the ceramic unit about 2000°K and that of the current-collecting edge, about 1600°K).

Figure 1b shows that the temperature distribution at the upper boundary of the block in contact with the hot gas differs substantially from that of Fig. 1a; the parts of the module near the copper edge have low surface temperatures ($T < 1000°K$), and such T imply that the electrical conductivity of a ZrO_2 module or the adjacent gas would be almost zero.

One expects that current leakage and breakdown between adjacent blocks would occur over gas gaps much thicker than those in Fig. 1a, which implies an increase in the effective size of the insulating gap between the hot conducting zones by comparison with the electrode of Fig. 1a.

These temperature distributions for ceramic modules show that a steel matrix is best used in a sectional MHD channel in which the axial electric field is weak.

Electrodes with copper matrices and current-collecting edges can be used in sectional channels with high axial electric fields (of Hall or diagonal type, etc.).

The solid lines in Fig. 2 show T_c and the temperature T_{me} of the cooling metal edge, as well as the temperature T_e at the current-collecting edge, for the case of the copper matrix; the dashed lines show the t dependence of T_c and T_{me} for the steel matrix case. The calculations were performed for $T_g = 3500°K$, $\alpha = 1000\ W/m^2 \cdot °K$ and $T_l = 300°K$ together with the dimensions chosen above.

It is clear that the time required to reach a steady-state temperature distribution is virtually the same for all the components in the block containing the copper matrix; however, this time is larger by about a factor of 1.5 than that for the case of steel.

An algorithm has been devised for calculating these two-dimensional temperature distributions, which was also used to determine the size of the electrode units for various T_g and α on the assumption of steady-state working.

Similarity theory allows one to represent the relative characteristic temperatures of the element as functions of the major definitive parameters:

$$\tilde{T}_c = \frac{T_c}{T_0} = \Phi_1\left\{\mathrm{Bi_s};\ \frac{L}{H};\ \frac{\delta_1}{H};\ \frac{\delta_2}{H},\ \frac{\delta_3}{H};\ \frac{T_g}{T_0};\ \frac{T_l}{T_0},\ \frac{\lambda_1}{\lambda_c};\ \frac{\lambda_2}{\lambda_c};\ \frac{\lambda_3}{\lambda_c}\right\},$$

$$\tilde{T}_{me} = \frac{T_{me}}{T_0} = \Phi_2\left\{\mathrm{Bi_s};\ \frac{L}{H};\ \ldots\right\},\ \tilde{T}_i = \frac{T_i}{T_0} = \Phi_3\left\{\mathrm{Bi_s};\ \frac{L}{H};\ \ldots\right\}, \quad (9)$$

$$\tilde{T}_e = \frac{T_e}{T_0} = \Phi_4\left\{\mathrm{Bi_s};\ \frac{L}{H};\ \ldots\right\}.$$

Here T_c, T_{me}, T_i, and T_e are the maximum temperatures at the hot surfaces of the ceramic module, matrix edge, insulating plate, and current-collecting edge, respectively, while T_g and $T_0 = T_c$ are, respectively, the gas temperature and the basic characteristic temperature scale factor; λ_c, λ_1, λ_2, and λ_3 are the thermal conductivity of the ZrO_2 ceramic, that of the cooling edge, that of the current-collecting edge, and that of the insulating insert; $\mathrm{Bi_s} = \alpha H/\lambda_c$ is the Biot number derived from λ_c for the ZrO_2.

We see from (9) that the sizes of the parts in the electrode block are completely determined by the relative values of the maximum temperatures $\tilde{T}_c$, $\tilde{T}_{me}$, $\tilde{T}_i$, $\tilde{T}_e$ for given T_g, T_l, and $\mathrm{Bi_s}$.

We have seen above that the parameters $\delta_1 \geq 1$-1.5 mm and $\delta_2 \geq 1$-1.5 mm can be used with the appropriate range in gas-flow parameters ($\mathrm{Bi_s} = 1.9$-3.1; $T_g/T_c = 1.5$-2.0) to leave considerable freedom of choice in the size of the cooling edges and ceramic insulating plate, since then there is little effect on the temperatures in the electrode block. Therefore, one can determine the relative width L/H and δ_3/H without considering δ_1/H and δ_2/H.

From (9) we then have for the above electrode designs and specified materials that

$$\frac{L}{H} = f_1\left(\mathrm{Bi_s},\ \frac{T_g}{T_c},\ \frac{T_e}{T_c},\ \frac{T_l}{T_c}\right);\ \frac{\delta_3}{H} = f_2\left(\mathrm{Bi_s},\ \frac{T_g}{T_c},\ \frac{T_e}{T_c},\ \frac{T_l}{T_c}\right).$$

The form of the relationship was derived from a large series of two-dimensional calculations on such units containing copper matrices and copper collectors and also ones with steel matrices. It was assumed that $T_c = 2000°K$, $T_e = 1600°K$, $T_l = 300°K$.

Figure 3 shows the results for ZrO_2 units with grid current collection and copper matrices.

It is clear that T_g/T_c has a marked effect on L/H (hyperbolic); however, it has little effect on δ_3/H within the relevant range of Bi_s and T_g, since this quantity is largely determined by Bi_s.

The numerical data can be approximated with reasonable precision by the following equations:

$$\frac{L}{H}=26.92Bi_s^{-1.2}\left(\frac{T_g}{T_c}\right)^{-3.13},\ \frac{\delta_3}{H}=[-0.0283+0.636\lg Bi_s]\left(\frac{T_g}{T_c}\right)^{0.1}. \quad (10)$$

Figure 4 shows L/H and Bi_s as functions of T_g/T_c for electrodes with steel matrices; the curves are of falling type, and the following functions provide a close fit for the range $T_g/T_c \sim 1.5-2$ for L/H as a function of T_g/T_c and Bi_s as a function of T_g/T_c:

$$\frac{L}{H}=7.44\left(\frac{T_g}{T_c}\right)^{-3}+0.418\left(\frac{T_g}{T_c}\right)-0.626,$$
$$Bi_s=4.28\left(\frac{T_g}{T_c}\right)^{-2}+0.5\left(\frac{T_g}{T_c}\right)-0.75. \quad (11)$$

Equations (10) and (11) allow one to define the sizes of the electrode blocks to provide a temperature of $T_c \cong 2000$°K for a ZrO_2 module for various Bi_s and T_g.

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PROPAGATION OF THERMAL DISTURBANCES IN MEDIA WITH VOLUMETRIC HEAT ABSORPTION

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UDC 536.24

The solution of one-dimensional unsteady problems of nonlinear heat conduction in the presence of temperature-dependent volumetric heat absorption in the medium is discussed. The conditions are found for the existence of generalized solutions describing temperature waves whose fronts propagate in the medium with a finite velocity.

The investigations carried out in [1, 2] made it possible to formulate the conditions under which the quasilinear equation of heat conduction

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